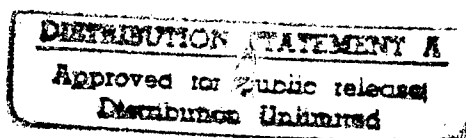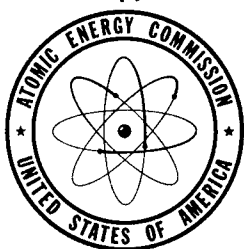NATIONAL SCIENCE FOUNDATION,
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Observations on Nonlinear Meson Dynamics

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It is well known that the possibility of the mutual transformation of particles results in effective nonlinearity of the equations of the corresponding fields. Nonlinear equations can be written in the form of relations of a phenomenological field theory, using quasi-dielectric permeabilities which may be characteristic of free space as well as of a real medium. Thus the nonlinear equation of a real scalar meson field in the presence of sources has the form [1]

$$\frac{\partial}{\partial x_\nu}\left(\varepsilon_1\frac{\partial\Phi}{\partial x_\nu}\right) - k_0^2\varepsilon_2\Phi = -4\pi g\rho, \tag{1}$$

where ϵ_1 and ϵ_2 are invariant functions, such as

$$\epsilon = 1 + a\Phi^2 + b\Phi_\nu^2, \tag{2}$$

as can be shown starting from the simplest nonlinear Lagrangian.

When it is sufficient to allow for ϵ_2, we have for an almost constant Φ in the static case

$$\Delta\Phi - k_0^2\Phi - \Lambda\Phi^3 = -4\pi g\rho. \tag{3}$$

Thus we obtain a nonlinear correction of the type that we used earlier[2] in the equations of relativistic quantum field theory. (Here we come to the nonlinear correction to the mass of particles determined basically by the term $\Lambda\Phi^3$.)

For small and nearly constant Φ's, Eq. (1) can be written in the form

$$\Delta\Phi - k_0^2\frac{\epsilon_2}{\epsilon_1}\Phi = -4\pi\frac{g}{\epsilon_1}\rho \approx -4\pi\rho g(1 - a^2\Phi^2 + \ldots), \tag{4}$$

which shows clearly the well-known equivalence of nonlinear coupling to the presence of a nonlinearity in the field equation. [3]

The difficult problem of determining the parameter Λ can be solved, for instance, by studying the nonlinear effect of the type involved in the scattering of mesons by mesons through nucleons or, similarly, in the scattering of light by light through electron-positron pairs[4] or through charged mesons. For this purpose, the matrix element of the above-mentioned 4th-order process due to the interaction of the nucleons with a meson field of the type $U = g\Phi$ must be compared with the matrix element for the same process due to the specific nonlinear interaction of mesons, represented by the following terms in the Lagrangian:

$$L' = \frac{1}{4\pi}\Lambda\Phi^4 + \beta\left(\frac{\partial}{\partial x_\nu}\Phi\right)^4. \tag{5}$$

Hence we have the estimate

$$\Lambda \cong \frac{g^4}{(\hbar c)^3}(f_0 + f_4), \tag{6}$$

where $f_0 = \int(dp)/(M^2c^2 + p^2)^{3/2}$, and f_4 is finite.

Consider now the right-hand side of the equation of the nonlinear field (3), determined by the density of nucleonic sources ρ. In the case of a system of nucleons, it is better to refine the usual empirical relation

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ρ = const, taking as a basis, for instance, the relation obtained by the Thomas-Fermi method, which, in the relativistic case, has the form[5]

$$\rho = \frac{1}{3\pi^2} \frac{g^3}{(\hbar c)^3} \left(\frac{2Mc^2}{g}\right)^{3/2} \Phi^{3/2} \left[1 + \frac{g\Phi}{2Mc^2}\right]^{3/2}.\qquad(7)$$

In this case we obtain, instead of (3),

$$\Delta\Phi - k_0^2\Phi - \Lambda\Phi^3 = -\frac{4}{3\pi} \frac{g^4}{(\hbar c)^3} \left(\frac{2Mc^2}{g}\right)^{3/2} \Phi^{3/2} \left[1 + \frac{g\Phi}{2Mc^2}\right]^{3/2}.\qquad(8)$$

If the ultrarelativistic approximation applies, we have

$$\Delta\Phi - k_0^2\Phi - \Lambda\Phi^3 = -4\pi g\rho = -\frac{4}{3\pi} \frac{g^4}{(\hbar c)^3} \Phi^3.\qquad(9)$$

If Φ varies slowly and the nonlinearity is strong ($k_0^2\Phi \ll \Lambda\Phi^3$), we find the curious result that the nonlinear parameter Λ can be determined. In fact, we have

$$-\Lambda\Phi^3 = -\frac{4}{3\pi} \frac{g^4}{(\hbar c)^3} \Phi^3,$$

and hence

$$\Lambda = \frac{4}{3\pi} \left(\frac{g^4}{\hbar^3 c^3}\right).\qquad(10)$$

This form of Λ agrees with the estimate obtained with the help of 4th-order effects. However, for actual nuclear systems, Eq. (10) gives too small a value because the ultrarelativistic approximation is too crude.

In the nonrelativistic limit we have

$$\Lambda\Phi^3 = 4\pi g\rho = \frac{4}{3\pi} \frac{g^4}{(\hbar c)^3} \left(\frac{2Mc^2}{g}\right)^{1/2} \Phi^{3/2}.\qquad(11)$$

If Λ is known, we obtain from (11) the following as the zeroth approximation:

$$\Phi_0 = \left(\frac{4}{3\pi\Lambda} \frac{g^4}{(\hbar c)^3}\right)^{2/3} \frac{2Mc^2}{g}.\qquad(12)$$

Conversely, writing Λ as

$$\Lambda = a \frac{g^4}{(\hbar c)^3}\qquad(13)$$

and adopting the empirical value $g\Phi_0 \cong 30$ Mev, we obtain $a \sim 2\cdot 10^2$. We note[6] that if $\rho = \text{const}\cdot\Phi^n$, the stability of nuclear systems requires that $n > 3/2$.

For the analysis of nonlinear mesodynamics, it is interesting to note that, in the case of slowly varying fields, a field equation of the type

$$\Phi'' + b\Phi^3 + c\Phi^2 + a\Phi + d = 0\qquad(14)$$

can be solved in elliptic functions, such as

$$\Phi = A + B \operatorname{cn}(\omega r + \gamma),\qquad(15)$$

where ω^2 and the coefficients A, B, and γ are determined by the parameters of the equation and by the initial conditions. From this solution we obtain in particular the trivial solution

$$\Phi_0 = \begin{cases} 0 \\ \pm\sqrt{-a/b}, \end{cases}\qquad(16)$$

for the case $c = d = 0$, when $\omega r + \gamma = (2n + 1)K$, where K is the complete elliptic integral of the first kind and n is an integer.

In the important case of Eq.(9), when $c = d = 0$, we have, in the presence of weak nonlinearity, $b < 0$ and $a < 0$; in other words, within certain limitations on the parameters of the solution, we obtain for large distances

the following solution from (15):

$$\Phi = \sqrt{\frac{2k_0^2}{\Lambda}} \left[\sinh(k_0 r + C_1)\right]^{-1} \qquad (C_1 = \text{const}). \qquad (17)$$

For $b = -\Lambda$, we have $a = -k_0^2$.

In the case of the equation $\Phi'' - \Lambda\Phi^3 = 0$, which corresponds to a strong nonlinearity, we obtain as a solution valid for small distances

$$\Phi = \sqrt{\frac{2}{\Lambda}} \frac{1}{r + \text{const}}. \qquad (18)$$

Analysis of the solutions obtained above and also of the role of ϵ shows clearly the weakening of the interactions transmitted by a nonlinear field over small distances, a weakening which seems definitely required for various empirical reasons.

The elliptic solutions correspond to the presence of a new parameter, which can be given the dimension of a length. Note that in Born-Infeld's nonlinear theory the potential was similarly expressed by an elliptic integral and that the nonlinear equation of Mie's electrodynamics, which essentially can be used only in meson dynamics, also had elliptic functions as solutions.

In conclusion, we express our thanks to N. N. Kolyosnikov for his valuable comments and for his participation in discussions.

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